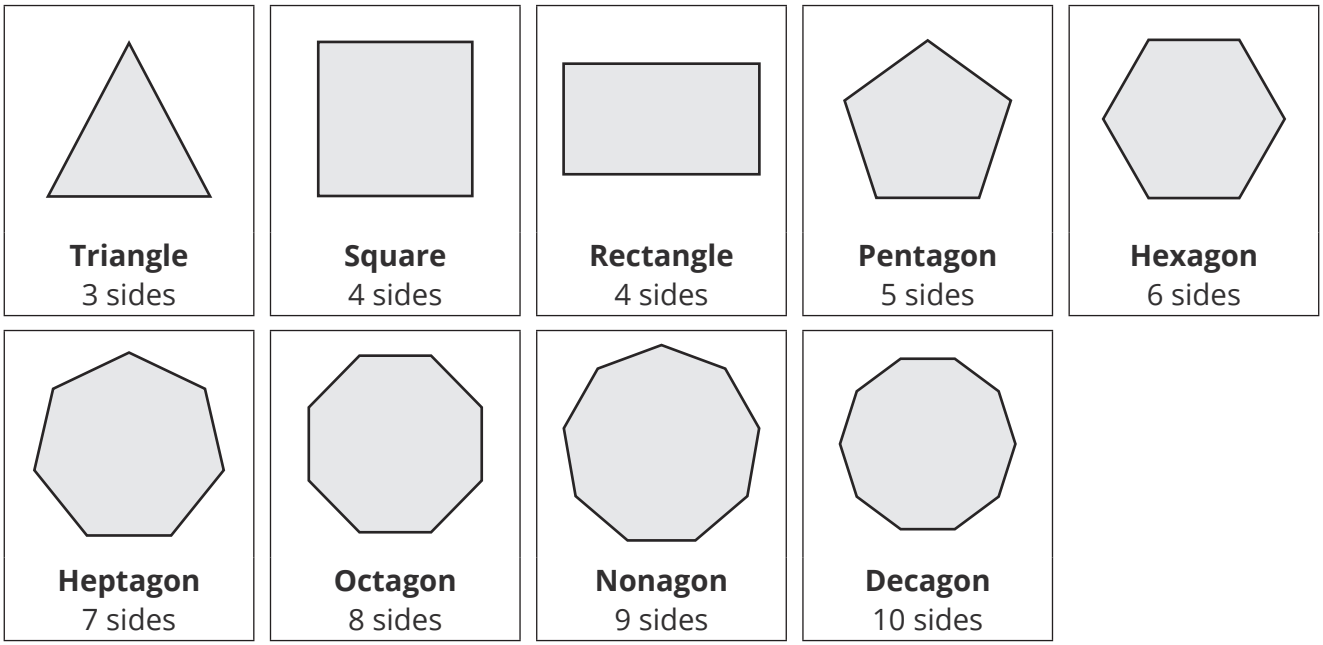
Angles in Polygons

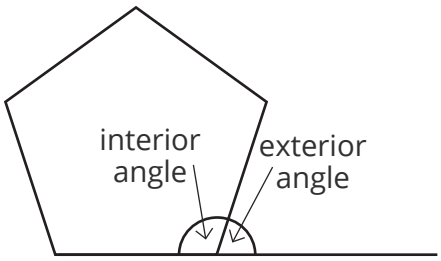
A **polygon** is a 2D shape which has **3 or more straight sides**.

In a regular polygon, all the sides are the **same length** and all the **angles** are the **same size**.

You must know the names of these **regular** polygons and how many **sides** they have.



You need to know how to find the **interior** and **exterior** angles of regular polygons.



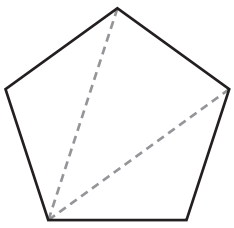
The **sum** of the **exterior** angles of a **polygon** is always 360°. In a **regular** polygon, to find an **exterior** angle, you can divide 360° by the number of sides ($\frac{360}{n}$).

An **interior** angle and its corresponding **exterior** angle add up to 180°.

The formula for the **sum** of the **interior** angles in a **polygon** is:

$(n - 2) \times 180°$ (where n is equal to the number of sides).

To understand the formula, divide a **regular** polygon into triangles.

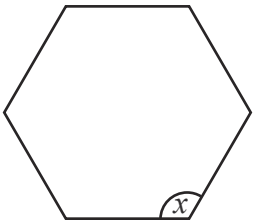


You can divide a regular pentagon into 3 triangles. The angles in a triangle add up to 180°. Notice that the number '3' is 2 less than the number of sides ($n - 2$).

There are 3 triangles, with the interior angles all adding up to 180° in each triangle. Hence, the formula $(n - 2) \times 180°$ or, in this case: $3 \times 180° = 540°$.

Example 1:

A regular hexagon is shown below. Calculate the missing angle marked x.



Use the formula to find the sum of the interior angles.

$(n - 2) \times 180°$

$(6 - 2) \times 180° = 720°$

As the hexagon is regular, all the interior angles are equal. Therefore, to find the size of the interior angle, divide the sum of the interior angles by the number of sides: 6.

$720 \div 6 = 120°$

$x = 120°$

Alternatively, the interior angle can be found by subtracting the exterior angle from 180°.

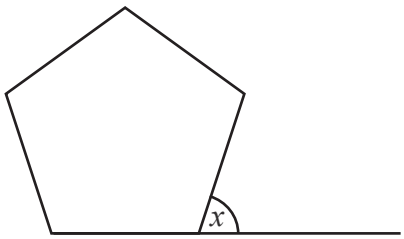
$360 \div 6 = 60°$ (the size of an exterior angle)

$180° - 60° = 120°$

$x = 120°$

Example 2:

A regular pentagon is shown below. Calculate the missing angle marked x.



The sum of the exterior angles for a polygon is 360°.

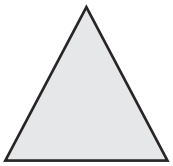
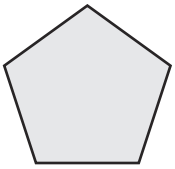
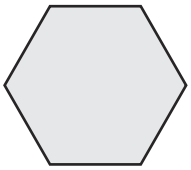
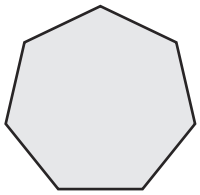
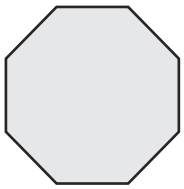
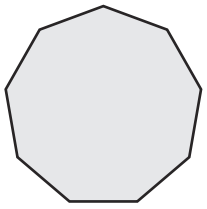
As the pentagon is regular, all the exterior angles are equal. Therefore, to find the missing angle, divide the sum of the exterior angles by the number of sides: 5.

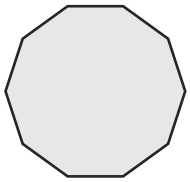
$360 \div 5 = 72°$

$x = 72°$

Your turn

1. Complete the table. The first one has been completed for you.
   Note: each polygon is regular.

Polygon	Sum of Interior Angles	Interior Angle	Exterior Angle
 **Triangle**	$(n - 2) \times 180°$ $(3 - 2) \times 180$ 180°	180 ÷ 3 60°	360 ÷ 3 120°
 **Rectangle**			
 **Pentagon**			
 **Hexagon**			
 **Heptagon**			
 **Octagon**			
 **Nonagon**			

 **Decagon**			

2. Work out the sum of the interior angles for a polygon with:

   a. 20 sides

   b. 45 sides

   c. 100 sides

3. The interior angles of a polygon add up to 2880°. Work out the number of sides the polygon has.

4. The interior angles of a polygon add up to 1980°. Work out the number of sides the polygon has.

5. The interior angles of a polygon add up to 3060°. Work out the number of sides the polygon has.

6. Calculate the size of each exterior angle in a regular polygon which has:

   a. 6 sides

   b. 10 sides

   c. 15 sides

   d. 20 sides

   e. 50 sides

7. A polygon has an exterior angle of 36°. Calculate the number of sides to the polygon.

8. A polygon has an interior angle of 175°. Calculate the number of sides to the polygon.

Challenge

A polygon has an interior angle that is five times larger than its exterior angle. How many sides does the polygon have?